RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2019-22]

B.A./B.Sc. FIRST SEMESTER (July – December) 2019

Mid-Semester Examination, September 2019

MATHEMATICS (Honours)

Date : 16/09/2019

Time : 11 am – 12 noon

Paper: I [CC 1]

Full Marks : 25

[Use a separate Answer Book for each group]

GROUP – A

Answer **any three** from question nos. 1 to 5 : (3 × 4)

1. Consider the following permutations in S_5,

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix} \text{and } \gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}.$$

Evalutate : $\alpha\beta$, $\alpha\gamma^2$, α^{-1} and $\gamma\alpha\gamma^{-1}$. [4]

2. Let $G = \{a, b, c, d\}$ be a group under the binary operation *. Complete the following Cayley table for the group: [4]

*	d	a	b	c
d	d			
a		c	d	
b				
c				

3. Let $f : A \to B$ be a function. Then show that f is injective iff $f(X \cap Y) = f(X) \cap f(Y)$ for all nonempty subsets X and Y of A. [4]

4. Find the remainder when $9.4^{24} + 2.9^{35}$ is divided by 5. [4]

5. Let R and L be two equivalence relations on a set A such that $R \circ L = L \circ R$. Show that $R \circ L$ is also an equivalence relation on A. [4]

GROUP – B

Answer question no. 6 and any two from the rest :

6. If $x, y \in \mathbb{R}$ with y>0, then prove that $\exists n \in \mathbb{N}$ such that $ny > x$.

or

If $a, b \in \mathbb{R}$, then show that

i) $\max\{a, b\} = \frac{1}{2}\{a + b + |a - b|\}$ and

ii) $\min\{a, b\} = \frac{1}{2}\{a + b - |a - b|\}$. [5]

7. Show that the intersection of a finite number of neighbourhoods of c is also a neighbourhood of c. [4]

8. Prove that a set W in $\mathbb{R}$ is open if and only if $W=W^0$, where W^0 is the interior of W. [4]

9. Show that every infinite bounded subset of $\mathbb{R}$ has at least one limit point in $\mathbb{R}$. [4]

10. Show that the derived set of any set in $\mathbb{R}$ is a closed set. [4]

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